ϕ = kernel of performance functional
$\varphi(x)$ = arbitrary function of x
$\psi(x)$ = arbitrary function of x

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Theoretical Development and Experimental Verification of a Novel, Well-Mixed Vessel

The mixing characteristics of a vessel containing no moving parts have been studied theoretically and experimentally. The vessel consists of two chambers separated by a porous barrier. Mixing results because elements of fluid permeating the barrier at various distances from the inlet reside for different periods of time within the vessel and combine with other elements having entered earlier and later. An apparatus was designed a priori and experimentally verified to give a residence-time distribution function the same as a completely mixed vessel. The method was extended to show that in principle a vessel exhibiting any residence-time distribution function can be designed by modifying the geometries of the chambers and the porous barrier.

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SCOPE

Well-mixed vessels are widely used in production, pilot plant, and research applications and in their most common form these systems consist of a container and an efficient stirrer. The theory and performance of such systems has been extensively studied and much attention has been given to testing how closely a given system approaches complete mixedness. One commonly applied test is to observe the transient tracer concentration in the effluent stream of a steady flow system when the tracer is introduced as a step-function in the input stream. This test permits the determination of the residence-time distribution function (RTDF) which can be used to evaluate how closely the performance of a given vessel approaches that of a completely-mixed vessel.

This paper concerns a new kind of mixing vessel which is novel in that it contains no moving parts. The new mixing concept appears to offer three distinct advantages over its older counterpart:

1. It eliminates the need for a stirrer-drive system and its attendant rotary seal.
2. It permits a priori design of any mixing behavior. More specifically, it can in principle be designed to give any predetermined RTDF.
3. It permits mixing without the dissipation of high mechanical energy inputs.

The basic principle upon which the new mixing vessel operates is to regulate the amount of flow through sections of a high-resistance porous barrier at progressively greater distances from the input and output of the vessel. In this way, the residence time of each fraction of the input stream can be controlled to predetermine the RTDF. The completely mixed vessel will be the main concern of this paper from the theoretical and experimental point of view; however, some effort will be made to generalize the theoretical treatment. Although the experimental work has been limited to gas-gas systems, the theory has no explicit limitations to gases and would apply to liquids. However, extremely high viscosities would probably invalidate the model.

It should be made clear at the outset that the completely-mixed vessel as obtained by a stirrer in a container is not mechanistically equivalent in its mixing properties to the completely mixed vessel obtained using the barrier. This derives from the well-known fact that two systems can display identical RTDF's but vary greatly in the detailed mechanisms which give rise to them. Therefore, the two systems are equivalent only with respect to first-order processes, that is, systems in which events depend uniquely upon time but not position in the vessel. Obviously, many systems do not meet this restriction. For example, the two vessels, although each had the same RTDF as a completely mixed vessel, would differ greatly in the conversion of a second-order chemical reaction for the same nominal residence time. This, perhaps, offers some interesting possibilities for the barrier mixer in a homogeneous or a heterogeneous reactor configuration. However, only first-order processes will be considered in this paper.

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CONCLUSIONS AND SIGNIFICANCE

This paper demonstrates that a mixer having no moving parts can be designed from first principles which gives a RTDF the same as for a completely stirred vessel. An apparatus based upon this design performed according to expectations when tested by transient tracer experiments in that it gave the characteristic exponential RTDF of a completely mixed vessel and the apparent volume obtained from the transient curve was within a few per cent of the measured volume. The excellent correspondence between design and actual behavior supports the contention that a vessel having any predetermined RTDF can in principle be designed by the generalized procedure.

The potential usefulness of a mixer without moving parts is significant when seals between the motor and stirrer would be exposed to high temperatures, high pressures, and/or corrosive fluids as might well be the case in chemical reaction systems.

Potential usefulness as a device to smooth temperature and concentration fluctuations on a time scale shorter than the nominal residence time are obvious.

THEORETICAL DEVELOPMENT

Figure 1 shows an idealized diagram of the mixer and represents the simplest form of the apparatus. There are two chambers, A and B, of equal cross-sectional area for flow S and equal volumes separated by a porous barrier. The flow enters chamber A and progresses in the $+x$ direction as elements flow continuously through the barrier into chamber B. As more elements pass through the barrier, the velocity in chamber A decreases as x increases. Simultaneously, elements flowing from chamber A into chamber B collect and flow toward the exit at $x = 0$. From a mass balance and the particular geometry chosen, the velocities in each chamber are equal at all values of x.

The length of time spent by an element in the vessel depends upon the distance the element travels before going through the barrier and the velocity in each of the chambers. In a qualitative sense, then, the RTDF is determined by the local permeability of the barrier. The objective, therefore, is to determine the permeability distribution $q(x)$ required to get a RTDF corresponding to a completely-mixed vessel. To determine this, the following assumptions are made:

1. The pressure drop across the porous wall is much greater than the pressure drop along the length of either chamber A or B; that is,

$$P_A(x) - P_B(x) >> \begin{cases} P_A(0) - P_A(L) \\ P_B(0) - P_B(L) \end{cases}$$

2. The absolute pressure in either chamber A or B is much greater than the pressure drop across the porous wall; that is,

$$P_A(x) \text{ or } P_B(x) >> P_A(x) - P_B(x)$$

3. The flow is one-dimensional in the x-direction in both chambers A and B.
4. At any level x in the vessel, the composition is uniform in each chamber.
5. Longitudinal diffusion is neglected.
6. The vessel is isothermal.
7. The operation is at steady state.

As shown in Figure 1, the barrier surface area for flow per unit length $A(x)$ is a constant so that the local mass flow rate through the barrier depends only on the local barrier permeability $q(x)$. The local mass flow rate through the barrier between x and $x + \Delta x$ is

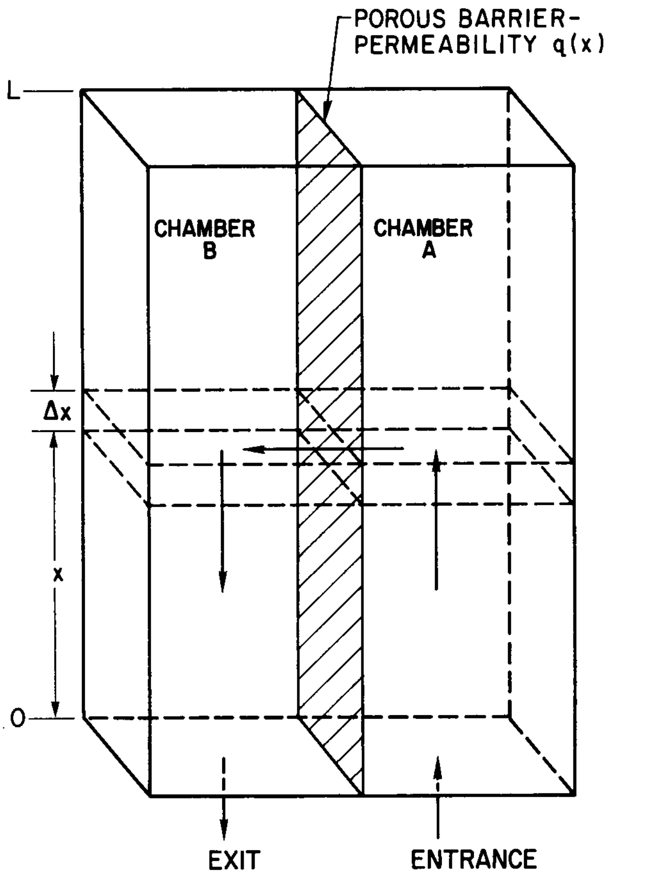
$$Aq(x)\,\Delta x \qquad (1)$$

From assumptions 1 and 2, the fluid is incompressible and the density ρ is constant throughout the vessel.

The objective is to find the local permeability function $q(x)$ of the porous wall such that the output from the vessel has the same response to mixing as an ideally stirred vessel. A response is characterized by the residence time distribution function $F(t)$ of the vessel. The RTDF is defined as the fraction of elements that enter at zero time which reside for time t or less in the vessel (Petersen, 1965). For the completely mixed vessel

$$F(t) = 1 - \exp\left(-\frac{vt}{V}\right) \qquad (2)$$

where v/V is the nominal residence time of the fluid. It also follows that the derivative of the RTDF is the fraction of the elements that enter at zero time which reside between a time t and $t + dt$ (Petersen, 1965). For the completely mixed vessel



ARROWS INDICATE ELEMENT OF FLUID THAT PASSES THRU THE BARRIER AT POSITION x

Fig. 1. Idealized porous barrier mixer.

$$\frac{d}{dt}[F(t)] = \frac{v}{V}\exp\left(-\frac{vt}{V}\right) \tag{3}$$

For the configuration under consideration (Figure 1), v is the total volumetric flow rate measured at $x = 0$ and $V = V_A + V_B$, that is, the total void volume of the reservoir.

From the assumption of plug flow in the x direction in both chambers A and B and instantaneous passage through the porous barrier, the time that an element of fluid resides in the reservoir is given by

$$t(x) = \int_0^x \frac{dx}{u_A(x)} + \int_x^0 \frac{dx}{-u_B(x)} = 2\int_0^x \frac{dx}{u(x)} \tag{4}$$

where $u(x) = u_A(x) = -u_B(x)$ is the velocity of the fluid in the x-direction. Equation (4) relates a position x with a specific residence time of fluid that passes through the barrier at x. Therefore, an element that passes through the barrier at $x = 0$ resides zero time and the element that passes through the barrier at $x = L$ resides an infinite time because $u(L) = 0$.

A material balance on the control volume $S\Delta x$ of chamber A on Figure 1 yields the expression

$$-\frac{1}{M(0)}\frac{dM(x)}{dx}\Delta x \tag{5}$$

which at steady state is equal to the fraction of the material entering at $x = 0$ that passes through the porous wall between x and $x + \Delta x$. Also from Equation (3) an expression for the fraction of material entering at $t = 0$ and residing for time between t and $t + \Delta t$ is

$$\frac{dF(t)}{dt}\Delta t = \left(\frac{v}{V}\exp\frac{-v}{V}t\right)\Delta t \tag{6}$$

Equating expression (5) and the right-hand side of Equation (6) and taking $\Delta x/\Delta t$ to the limit yields

$$-\frac{1}{M(0)}\frac{dM(x)}{dx}\left(\frac{dx}{dt}\right) = \frac{v}{V}\exp\left(-\frac{v}{V}t\right) \tag{7}$$

which upon substitution of Equation (4) gives

$$-\frac{1}{M(0)}\frac{dM(x)}{dx}\left(\frac{u(x)}{2}\right) = \left(\frac{v}{V}\right)\exp\left[-\frac{2v}{V}\int_0^x \frac{dx}{u(x)}\right] \tag{8}$$

an ordinary integro-differential equation.

The velocity $u(x)$ is related in a simple way to $M(x)$ by

$$M(x) = \rho S u(x) \tag{9}$$

Therefore, substitution and rearrangement yields

$$-\frac{M(x)}{2\rho S M(0)}\frac{dM(x)}{dx} = \frac{v}{V}\exp\left[-\frac{2v\rho S}{V}\int_0^x \frac{dx}{M(x)}\right] \tag{10}$$

Define $z \equiv \frac{x}{L}$ and $\phi \equiv \frac{M(x)}{M(0)}$ and note that $2SL = V$ and $\rho v = M(0)$. Substituting these dimensionless quantities into Equation (10) yields

$$\phi(z)\frac{d\phi(z)}{dz} = -\exp\left[-\int_0^z \frac{dz}{\phi(z)}\right] \tag{11}$$

having the solution

$$\phi(z) = \exp\left[-\int_0^z \frac{dz}{\phi(z)}\right] + \alpha \tag{12}$$

The constant of integration α equals zero from the boundary condition that $\phi(0) = 1$.

Equations (11) and (12) yield

$$\frac{d\phi(z)}{dz} = -1 \tag{13}$$

which upon integration gives

$$\phi(z) = -z + \beta$$

The integration constant $\beta = 1$ from the boundary condition that $\phi(1) = 0$. Therefore, the solution to Equation (11) becomes

$$\phi(z) = 1 - z \tag{14}$$

From Equations (1) and (5)

$$-\frac{dM(x)}{dx} = Aq(x) \tag{15}$$

However, from the definitions of ϕ and z and Equation (13)

$$-\frac{dM(x)}{dx} = -\frac{M(0)}{L}\frac{d\phi(z)}{dz} = \frac{M(0)}{L} \tag{16}$$

Hence to obtain a RTDF corresponding to a completely mixed vessel the permeability should be

$$q(x) = \frac{M(0)}{AL} = \text{constant} \tag{17}$$

In summary, then, Equation (17) means that a uniform local permeability over the length of the porous plate separating chamber A and B gives rise to a RTDF for this configuration that is equal to the response of a completely mixed vessel of the same void volume.

GENERALIZATION OF THE MODEL TO AN ARBITRARY GEOMETRY

Relaxation of the constraints on uniform cross section would make application of the design equation more flexible. This is considered now.

Let $S_A(x)$ and $S_B(x)$ be the cross-sectional areas at x of chambers A and B, respectively. Modification of Equation (9) yields

$$M(x) = \rho S_A(x)u_A(x) = -\rho S_B(x)u_B(x) \tag{18}$$

where the negative sign is necessary as before to account for the direction of flow's being opposite in chambers A and B. This modifies Equation (4) such that

$$t(x) = \int_0^x \frac{\rho S_A(x)}{M(x)}dx - \int_x^0 \frac{\rho S_B(x)}{M(x)}dx \tag{19}$$

Substitution of Equation (19) into Equation (7) yields upon simplification

$$-\frac{1}{M(0)}\frac{dM(x)}{dx}\left[\frac{M(x)}{\rho[S_A(x) + S_B(x)]}\right] = -\frac{v}{V}\exp\left[-\frac{v}{V}\rho\int_0^x \frac{S_A(x) + S_B(x)}{M(x)}dx\right] \tag{20}$$

Define

$$dz \equiv \left[\frac{S_A(x) + S_B(x)}{V}\right]dx$$

and

$$\phi \equiv \frac{M(x)}{M(0)}$$

and remember that $\rho v = M(0)$, then Equation (20) becomes

$$\phi \frac{d\phi}{dz} = -\exp\left[-\int_0^z \frac{dz}{\phi}\right] \quad (21)$$

which is identical to Equation (11) and of course has the same boundary conditions. The solution is, therefore, the same. Hence,

$$\phi(z) = 1 - z \quad (22)$$

As before, from Equations (1) and (5)

$$-\frac{dM}{dx} = A(x)q(x) \quad (23)$$

where we now generalize by allowing $A(x)$ to also vary with position. That is, for a general geometry, $A \neq$ constant. For example, a conical shaped barrier would have $A \propto x$.

Now solve for $-dM/dx$ in Equation (22) and equate to Equation (23). The result is

$$q(x)A(x) = \frac{M(0)}{V}[S_A(x) + S_B(x)] \quad (24)$$

A RTDF for a completely mixed vessel obtains from a multitude of shapes of barriers and passages so long as Equation (24) is satisfied.

As a practical matter $q(x)$, the permeability, would probably be made constant under all circumstances because the manufacture of a barrier with variable permeability would be difficult and costly. Furthermore, to obtain a RTDF corresponding to a completely mixed vessel we undoubtedly would choose q, A, S_A, and S_B all constants. However, if one sought a different RTDF, then certainly it would be necessary to let A, S_A, and S_B vary with length. To determine what the variation should be, a solution to an equation formed by the left-hand side of Equation (20) equated to the derivative of the desired RTDF curve would have to be generated. In general this could be difficult.

Although less elegant, an alternative approach would be to invert the problem as follows. Choose $q(x)$, $A(x)$, $S_A(x)$, and $S_B(x)$. Using Equations (19) and (25) below, solve for the corresponding derivative of the RTDF function.

$$\frac{q(x)A(x)}{\rho M(0)S_A(x)} \int_0^x q(x)A(x)dx = \begin{array}{c}\text{Derivative}\\ \text{of RTDF}\end{array} \quad (25)$$

Since the solution involves a quadrature it is straightforward even if numerical integration would be necessary.

Therefore it has been shown that this vessel can be employed as a well-mixed vessel or as any other mixer depending on the choice of $q(x)$, $A(x)$, $S_A(x)$, and $S_B(x)$.

EXPERIMENTAL VERIFICATION OF THE WELL-MIXED VESSEL

A practical design having cylindrical symmetry was developed which consisted of an outer cylinder and a inner porous cylinder. The two cylinders separated the unit into an inner chamber A and an outer chamber B. The inlet and outlet were at the same end of the test unit. The RTDF was determined experimentally using a transient tracer step-input.

A schematic diagram of the experimental apparatus used for the tracer experiments is shown in Figure 2. Helium was used as the main stream carrier gas and reference gas in the thermal conductivity (T.C.) cell. This allowed only the change of the nitrogen tracer concentration with time to be recorded. Preliminary tests showed that the T.C. cell response was linear with increasing nitrogen concentration over the nitrogen and helium flow rates used in the experiments. The flow rate of helium and nitrogen were measured with a calibrated rotameter and soap bubble meter, respectively. Prior to taking data the major pressure assumptions were investigated over the flow range of the experiments. A differential water manometer confirmed that assumptions 1 and 2 were easily satisfied.

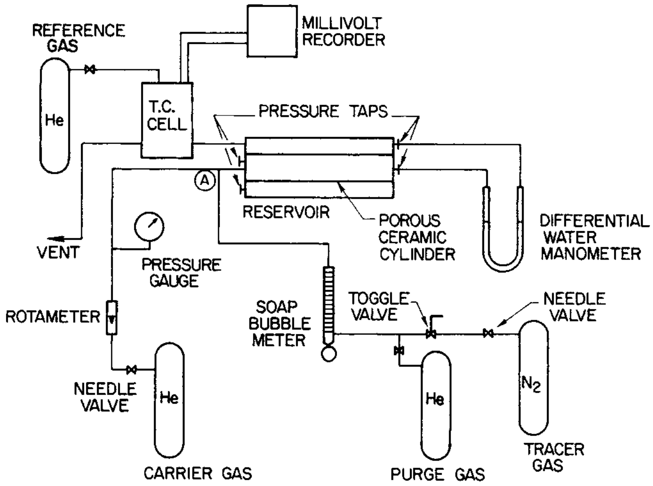


Fig. 2. Schematic diagram of apparatus used for tracer studies of the porous barrier vessel.

The line from the toggle valve to junction A was purged of all nitrogen with helium before each experiment. Once the toggle valve was switched to the open position, a volume of helium was transported ahead of the nitrogen at the same flow rate as the tracer step. This was necessary so that the baseline change induced by the additional flow of the tracer stabilized prior to the tracer's entering the reservoir. In this way the initial response of the tracer step was not obscured by the response of the recorder to the flow change. This precaution is essential to the early time interpretation of the data and the determination of zero time for tracer entrance. The assumption of an ideal step-function was evaluated through bypassing the reservoir and was found to be justified experimentally.

An unglazed ceramic cylinder 7.6 cm in diam. by 0.48 cm thick by 29.2 cm long was used for the porous cylindrical barrier. Preliminary testing showed that the cylinder was uniformly porous. The total void volume of the reservoir was approximately 2468 cm^3. Chambers A and B were approximately equal in cross-sectional area.

METHOD OF EXPERIMENTAL ANALYSIS

The behavior of an ideally stirred vessel follows the well-known exponential relationship

$$1 - \frac{C_A}{C_{A0}} = \exp\left(-\frac{v}{V}t\right) \quad (26)$$

where C_A is the concentration of tracer measured at the outlet at any time t.

The experimental data are evaluated by comparing a calculated apparent volume derived from the slope of a semi-log plot of $(1 - C_A/C_{A0})$ versus t with the actual geometric volume of the reservoir. Therefore the two criteria that must be met by the data are that they be correlated by an exponential function and that the time constant be the same as for an ideally mixed vessel.

A few preliminary experiments were performed without the inner porous cylinder to ascertain its behavior. The data indicated that without the porous barrier the vessel was not a well-mixed vessel.

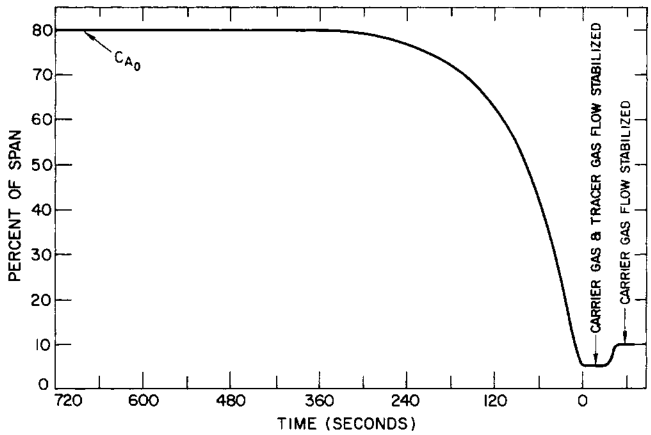


Fig. 3. Typical response curve to a step input of nitrogen for the well-mixed porous barrier vessel.

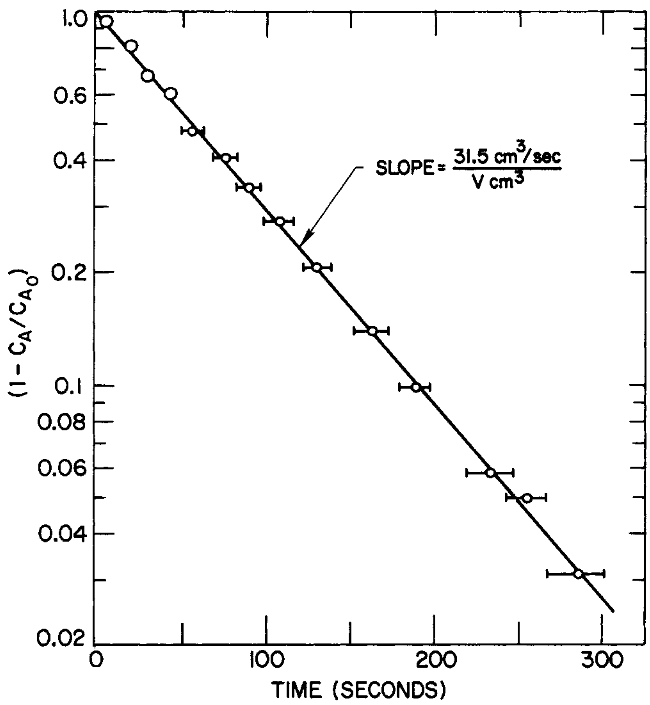


Fig. 4. Typical correlation of step input response for the well-mixed porous barrier vessel.

EXPERIMENTAL RESULTS

The objectives of the experiments were (1) to determine whether the uniform porous barrier reservoir did in fact have a RTDF the same as a well-stirred vessel, and (2) to investigate some physical phenomena that may cause deviation from the exponential behavior of the ideally stirred vessel.

A typical response curve is shown in Figure 3. The extracted data is plotted on semi-log coordinates in Figure 4 to illustrate the excellent exponential correlation of the data.

The calculated apparent volume derived from experiments performed over a wide range of flow rates are tabulated in Table 1. Alhough there is some scatter in the calculated values, the average value from the data is 2410 cm^3, which corresponds within 2.5% of the true geometric volume of 2468 cm^3. This deviation is well within any experimental certainty claimed for the data. On this basis it is concluded that the model developed theoretically is a physically realizable apparatus having the same RTDF as an ideally mixed reservoir of the same volume.

Other phenomena were also investigated. In the above experiments, sintered metal plates were placed in the entrance and exit end plate of the reservoir which served to distribute the entrance and exit flows uniformly over the cross-sectional area to avoid any jet type entrance effects (Schlichting, 1955; Squire, 1953). The sintered plates were then removed to investigate the effect of an entrance 1.37 cm (¼-in. NPT) orifice without aid of the distributors.

Table 2 shows the results. The calculated average apparent volume is 2050 cm^3, which is a 16% decrease from the geometric volume of the reservoir. The trend in the data suggests that as the volumetric flow rates increased the deviation from the true volume increased, which is consistent with a jet entrance effect.

The Reynolds number for the flow in chamber *A* in the above experiments is of the order of 10, and accordingly the entrance length would be about 0.1 of the reservoir length if the walls were nonporous. However, the manifold effect strongly flattens the velocity profile as shown by Berman (1953) and the profile more closely approximates plug flow than parabolic flow. Without the distributor, the flow jets into the chamber and the flow no longer approximates plug flow and assumption 3 is invalidated.

In an attempt to investigate experimentally the effects of velocity profiles, ceramic 9.5 mm (⅜-in.) spheres were alternately placed in chambers *A* and *B*. The effect of putting the spheres into the chambers would be (1) to flatten the velocity profile to a greater extent than without the spheres and (2) to increase the superficial velocity in the chamber with the spheres. Of course the

TABLE 1. SUMMARY OF DATA FOR TRACER STUDIES PERFORMED ON THE WELL-MIXED POROUS BARRIER VESSEL

Run no.	Helium carrier gas flow rate, cm^3/s	Nitrogen carrier gas flow rate, cm^3/s	Experimental apparent volume, cm^3
PP6	25.0	2.6	2,480
PP5	28.8	2.7	2,470
KK4	36.1	1.65	2,460
EE1	36.1	1.7	2,580
EE2	42.9	2.1	2,330
FF1	42.9	2.5	2,540
PP1	51.5	2.9	2,360
EE4	51.5	2.9	2,340
KK2	51.5	3.0	2,290
PP2	55.6	2.4	2,260
PP3	60.3	4.2	2,330
KK3	66.2	6.3	2,770
PP4	69.9	6.9	2,120

TABLE 2. SUMMARY OF DATA FOR TRACER STUDIES PERFORMED ON THE POROUS BARRIER VESSEL WITHOUT ENTRANCE AND EXIT FLOW DISTRIBUTORS

Run no.	Helium carrier gas flow rate, cm^3/s	Nitrogen tracer gas flow rate, cm^3/s	Experimental apparent volume, cm^3
AA3	42.9	2.2	2,080
SS0	51.5	3.1	2,310
DD2	66.2	6.7	2,200
AA4	79.4	7.7	1,850
AA5	79.4	7.7	1,770

Table 3. Summary of Data for Tracer Studies Performed on the Porous Barrier Vessel Packed with Ceramic Spheres

Run no.	Helium carrier gas flow rate, cm^3/s	Nitrogen tracer gas flow rate, cm^3/s	Experimental apparent volume, cm^3
I. With spheres packed into chamber A			
NN4	21.8	2.25	2,080
NN1	36.1	1.50	2,060
NN2	51.5	3.0	2,150
NN3	66.2	6.7	1,990
II. With spheres packed into chamber B			
MM2	36.1	1.6	2,310
MM1	51.5	2.9	2,290
MM3	64.2	5.3	1,980

geometric volume is reduced by the space occupied by the solid spheres. As before the sintered distributing plates were used at the entrance and exit.

The results of the experiments are presented in Table 3. The geometric volume with spheres in chamber A was approximated to be 1820 cm^3 and with spheres in chamber B to be 1980 cm^3. The average calculated apparent volumes abstracted from the experimental data are 2070 cm^3 and 2190 cm^3, respectively. The experimental data correlated well as exponential functions. Even though the experimentally derived volumes are higher than the approximated geometric volumes, the errors in determining void volumes are larger for these experiments. The volume occupied by the spheres was approximated by (1) displacement of water and (2) approximating the diameter of the irregular spheres as 0.95 cm. The values reported are an average of the two methods which themselves deviated by 5%. Therefore, it is difficult to determine if the deviation of the apparent volume from the geometric volume is a result of experimental errors or changes in flow properties caused by the spheres. It is also possible that cross-sectional areas S_A and S_B in the two experiments are functions of x due to the effect of the wall on the local packing density of the relatively large spheres. Equation (24) shows that such deviations could invalidate the method of interpretation slightly. Nevertheless, the relative changes in experimental apparent volume are consistent with the changes in calculated geometric volumes and the model appears to approximate the experiments quite well.

ADDITIONAL COMMENTS

A theoretical development and experimental verification of a novel porous barrier well-mixed reservoir has been presented. However, several precautions were required to ensure that the apparatus behaved according to the model. The need for entrance and exit distributors to eliminate entrance effects has been experimentally demonstrated. In designing a reservoir of this kind, chamber A can be viewed as a uniformly distributed manifold and the momentum of the incoming stream and pressure drops both across the barrier and down the chamber must be considered (Berman, 1953; Acrivos, 1959; Grobman et al., 1957; Senecal, 1957).

The concentric cylinder configuration presented should not be construed as the only physical realization of the reservoir. Any configuration that can satisfy the assumptions is applicable. The generalization of the equations to include varying cross-sectional areas for flow normal to and through the barrier suggest an infinitude of RTDF behavior with varying configurations.

The application of this type of vessel as a heterogeneous or homogeneous catalyzed reactor is as yet unexplored. For a first-order reaction the conversion would be the same as for a CSTR (Danckwerts, 1958; Zwietering, 1959), but in the other cases differences would be expected. Kramers and Westerterp (1963) treated the somewhat analogous problem of the ideal cross-flow reactor and found that for one case it proved to yield a better selectivity than either the plug flow reactor or the continuous flow stirred tank reactor. It could be expected that similar examples may be found wherein the present configuration would prove to be a significant improvement over the performance of more conventional reactors.

NOTATION

$A(x)$ = geometric function of porous barrier surface area/unit length evaluated at x, cm^2/cm
$C_A(t)$ = concentration of tracer in outlet of reservoir
C_{A_0} = tracer step concentration entering at time zero
$M(x)$ = mass flow rate in x direction at position x, g/s
$P_A(x)$, $P_B(x)$ = absolute pressure at position x in chambers A and B respectively
$q(x)$ = local permeability function, flux of fluid through the barrier at x, g/cm^2-s
$S_A(x)$, $S_B(x)$ = cross-sectional area at x of chamber A and B respectively, cm^2
t = time, s
$u_A(x)$, $u_B(x)$ = velocity in the x direction at position x in chamber A and B respectively, cm/s
v = total volumetric flow rate entering reservoir at $x = 0$, cm^3/s
V = total void volume of reservoir, cm^3
V_A, V_B = volume of chamber A and B respectively, cm^3
x = position along the length of porous barrier, cm
$z \equiv x/L$ = dimensionless position along length of barrier

Greek Letters

α = constant of integration
β = constant of integration
$\phi(x) \equiv M(x)/M(0)$ = dimensionless mass flow rate in x direction at position x
ρ = density of fluid in reservoir, g/cm^3

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